

ON SOME INTEGRAL INEQUALITIES FOR s -GEOMETRICALLY CONVEX FUNCTIONS AND THEIR APPLICATIONS

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ABSTRACT. In this paper, we establish three inequalities for differentiable s -geometrically and geometrically convex functions which are connected with the famous Hermite-Hadamard inequality holding for convex functions. Some applications to special means of positive real numbers are given.

1. INTRODUCTION

In this section we will present definitions and some results used in this paper.

Definition 1. Let I be an interval in \mathbb{R} . Then $f : I \rightarrow \mathbb{R}, \emptyset \neq I \subseteq \mathbb{R}$ is said to be convex if

$$(1.1) \quad f(tx + (1-t)y) \leq tf(x) + (1-t)f(y).$$

for all $x, y \in I$ and $t \in [0, 1]$.

Definition 2. [1] Let $s \in (0, 1]$. A function $f : I \subset \mathbb{R}_0 = [0, \infty) \rightarrow \mathbb{R}_0$ is said to be s -convex in the second sense if

$$(1.2) \quad f(tx + (1-t)y) \leq t^s f(x) + (1-t)^s f(y)$$

for all $x, y \in I$ and $t \in [0, 1]$.

It can be easily checked for $s = 1$, s -convexity reduces to the ordinary convexity of functions defined on $[0, \infty)$.

Recently, In [2], the concept of geometrically and s -geometrically convex functions was introduced as follows.

Definition 3. [2] A function $f : I \subset \mathbb{R}_+ = (0, \infty) \rightarrow \mathbb{R}_+$ is said to be a geometrically convex function if

$$(1.3) \quad f(x^t y^{1-t}) \leq [f(x)]^t [f(y)]^{1-t}$$

for all $x, y \in I$ and $t \in [0, 1]$.

Definition 4. [2] A function $f : I \subset \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is said to be a s -geometrically convex function if

$$(1.4) \quad f(x^t y^{1-t}) \leq [f(x)]^{t^s} [f(y)]^{(1-t)^s}$$

for some $s \in (0, 1]$, where $x, y \in I$ and $t \in [0, 1]$.

Date: October 08, 2012.

2000 Mathematics Subject Classification. Primary 26D10, 26D15.

Key words and phrases. geometrically convex, s -geometrically convex, hölder inequality, power mean inequality.

This paper is in final form and no version of it will be submitted for publication elsewhere.

If $s = 1$, the s -geometrically convex function becomes a geometrically convex function on \mathbb{R}_+ .

Example 1. [2] Let $f(x) = x^s/s$, $x \in (0, 1]$, $0 < s < 1$, $q \geq 1$, and then the function

$$(1.5) \quad |f'(x)|^q = x^{(s-1)q}$$

is monotonically decreasing on $(0, 1]$. For $t \in [0, 1]$, we have

$$(1.6) \quad (s-1)q(t^s - t) \leq 0, \quad (s-1)q((1-t)^s - (1-t)) \leq 0.$$

Hence, $|f'(x)|^q$ is s -geometrically convex on $(0, 1]$ for $0 < s < 1$.

In [4], the following Lemma and its related Hermite-Hadamard type inequalities for convex functions were obtained.

Lemma 1. [4] Let $f : I^\circ \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° , $a, b \in I^\circ$ with $a < b$. If $f' \in L[a, b]$, then the following equality holds:

$$(1.7) \quad \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx = \frac{b-a}{2} \int_0^1 (1-2t) f'(ta + (1-t)b) dt.$$

Theorem 1. [4] Let $f : I^\circ \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° , $a, b \in I^\circ$ with $a < b$. If $|f'|$ is convex on $[a, b]$, then the following inequality holds:

$$(1.8) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{(b-a)(|f'(a)| + |f'(b)|)}{8}.$$

Theorem 2. [4] Let $f : I^\circ \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I° , $a, b \in I^\circ$ with $a < b$, and let $p > 1$. If the mapping $|f'|^{p/(p-1)}$ is convex on $[a, b]$, then the following inequality holds:

$$(1.9) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{2(p+1)^{1/p}} \left[\frac{|f'(a)|^{p/(p-1)} + |f'(b)|^{p/(p-1)}}{2} \right]^{(p-1)/p}.$$

The goal of this paper is to establish some inequalities of Hermite-Hadamard type for geometrically and s -geometrically convex functions.

2. ON SOME INEQUALITIES FOR s -GEOMETRICALLY CONVEXITY

Theorem 3. Let $f : I \subseteq \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be a differentiable mapping on I° , $a, b \in I^\circ$ with $a < b$, and $f' \in L[a, b]$. If $|f'|$ is s -geometrically convex and monotonically decreasing on $[a, b]$ for $s \in (0, 1]$, then the following inequality holds:

$$(2.1) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{2} G_1(s; g_1(\alpha), g_2(\alpha))$$

where

$$(2.2) \quad g_1(\alpha) = \begin{cases} \frac{1}{4} & \alpha = 1 \\ \frac{2\alpha^{1/2} - 2 - \ln \alpha}{(\ln \alpha)^2} & \alpha \neq 1 \end{cases}, \quad g_2(\alpha) = \begin{cases} \frac{1}{4} & \alpha = 1 \\ \frac{2\alpha^{1/2} - 2\alpha + \alpha \ln \alpha}{(\ln \alpha)^2} & \alpha \neq 1 \end{cases}$$

$$\alpha(u, v) = |f'(a)|^u |f'(b)|^{-v}, \quad u, v > 0,$$

$$G_1(s; g_1(\alpha), g_2(\alpha)) = |f'(b)|^s [g_1(\alpha(s, s)) + g_2(\alpha(s, s))], \quad |f'(a)| \leq 1.$$

Proof. Since $|f'|$ is s -geometrically convex and monotonically decreasing on $[a, b]$, from Lemma 1, we have

$$\begin{aligned}
 & \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \\
 & \leq \left| \frac{b-a}{2} \int_0^1 (1-2t) f'(ta + (1-t)b) dt \right| \\
 & \leq \frac{b-a}{2} \int_0^1 |1-2t| |f'(ta + (1-t)b)| dt \\
 & \leq \frac{b-a}{2} \left\{ \int_0^{\frac{1}{2}} (1-2t) |f'(a^t b^{1-t})| dt + \int_{\frac{1}{2}}^1 (2t-1) |f'(a^t b^{1-t})| dt \right\} \\
 & \leq \frac{b-a}{2} \left\{ \int_0^{\frac{1}{2}} (1-2t) |f'(a)|^{t^s} |f'(b)|^{(1-t)^s} dt + \int_{\frac{1}{2}}^1 (2t-1) |f'(a)|^{t^s} |f'(b)|^{(1-t)^s} dt \right\}.
 \end{aligned}$$

If $0 < \mu \leq 1$, $0 < \alpha, s \leq 1$, then

$$(2.3) \quad \mu^{\alpha^s} \leq \mu^{\alpha s}.$$

If $|f'(a)| \leq 1$, by (2.3), we get that

$$\begin{aligned}
 & \int_0^{\frac{1}{2}} (1-2t) |f'(a)|^{t^s} |f'(b)|^{(1-t)^s} dt + \int_{\frac{1}{2}}^1 (2t-1) |f'(a)|^{t^s} |f'(b)|^{(1-t)^s} dt \\
 & \leq \int_0^{\frac{1}{2}} (1-2t) |f'(a)|^{st} |f'(b)|^{s(1-t)} dt + \int_{\frac{1}{2}}^1 (2t-1) |f'(a)|^{st} |f'(b)|^{s(1-t)} dt \\
 & = \int_0^{\frac{1}{2}} (1-2t) |f'(b)|^s \left| \frac{f'(a)}{f'(b)} \right|^{st} dt + \int_{\frac{1}{2}}^1 (2t-1) |f'(b)|^s \left| \frac{f'(a)}{f'(b)} \right|^{st} dt \\
 (2.4) \quad & |f'(b)|^s [g_1(\alpha(s, s)) + g_2(\alpha(s, s))]
 \end{aligned}$$

Thus, immediately gives the required inequality (2.1). \square

Theorem 4. Let $f : I \subseteq \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be a differentiable mapping on I° , $a, b \in I^\circ$ with $a < b$, and $f' \in L[a, b]$. If $|f'|^q$ is s -geometrically convex and monotonically decreasing on $[a, b]$ for $1/p + 1/q = 1$ and $s \in (0, 1]$, then the following inequality holds:

$$(2.5) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{2^{(p+1)^{1/p}}} G_2(s, q; g_3(\alpha))$$

where

$$(2.6) \quad g_3(\alpha) = \begin{cases} 1, & \alpha = 1, \\ \frac{\alpha-1}{\ln \alpha}, & \alpha \neq 1, \end{cases}$$

$$(2.7) \quad G_2(s, q; g_3(\alpha)) = |f'(b)|^s [g_3(\alpha(sq, sq))]^{\frac{1}{q}}, \quad |f'(a)| \leq 1.$$

Proof. Since $|f'|^q$ is s -geometrically convex and monotonically decreasing on $[a, b]$, from Lemma 1 and Hölder inequality, we have

$$\begin{aligned}
 (2.8) \quad & \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \\
 & \leq \frac{b-a}{2} \int_0^1 |1-2t| |f'(ta + (1-t)b)| dt \\
 & \leq \frac{b-a}{2} \left(\int_0^1 |1-2t|^p dt \right)^{\frac{1}{p}} \left(\int_0^1 |f'(ta + (1-t)b)|^q dt \right)^{\frac{1}{q}}
 \end{aligned}$$

Using the properties of $|f'|^q$, we obtain that

$$\begin{aligned}
 (2.9) \quad & \left(\int_0^1 |f'(ta + (1-t)b)|^q dt \right)^{\frac{1}{q}} \leq \left(\int_0^1 |f'(a^t b^{1-t})|^q dt \right)^{\frac{1}{q}} \\
 & \leq \left(\int_0^1 |f'(a)|^{qt^s} |f'(b)|^{q(1-t)^s} dt \right)^{\frac{1}{q}}.
 \end{aligned}$$

If $|f'(a)| \leq 1$, by (2.3), we get that

$$\begin{aligned}
 (2.10) \quad & \left(\int_0^1 |f'(a)|^{qt^s} |f'(b)|^{q(1-t)^s} dt \right)^{\frac{1}{q}} \leq \left(\int_0^1 |f'(a)|^{sq t} |f'(b)|^{sq(1-t)} dt \right)^{\frac{1}{q}} \\
 & = \left(|f'(b)|^{sq} \int_0^1 \left| \frac{f'(a)}{f'(b)} \right|^{sq t} dt \right)^{\frac{1}{q}} \\
 & = |f'(b)|^s [g_3(\alpha(sq, sq))]^{\frac{1}{q}}.
 \end{aligned}$$

Further, since

$$(2.11) \quad \int_0^1 |1-2t|^p dt = \int_0^{\frac{1}{2}} (1-2t)^p dt + \int_{\frac{1}{2}}^1 (2t-1)^p dt = 2 \int_0^{\frac{1}{2}} (1-2t)^p dt = \frac{1}{p+1}$$

a combination of (2.8)-(2.11) immediately gives the proof of inequality (2.5). \square

Corollary 1. Let $f : I \subseteq (0, \infty) \rightarrow (0, \infty)$ be differentiable on I° , $a, b \in I$ with $a < b$, and $f' \in L([a, b])$. If $|f'|^q$ is s -geometrically convex and monotonically decreasing on $[a, b]$ for $s \in (0, 1]$, then

i) When $p = q = 2$, one has

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{2\sqrt{2}} G_2(s, 2, g_3(\alpha))$$

ii) If we take $s = 1$ in (2.5), we have for geometrically convex, one has

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{2(p+1)^{\frac{1}{p}}} G_2(1, q, g_3(\alpha))$$

where g_3, G_2 are same with (2.6), (2.7).

Theorem 5. Let $f : I \subseteq \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be a differentiable mapping on I° , $a, b \in I^\circ$ with $a < b$, and $f' \in L[a, b]$. If $|f'|^q$ is s -geometrically convex and monotonically decreasing on $[a, b]$ for $q \geq 1$ and $s \in (0, 1]$, then the following inequality holds:

$$(2.12) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{2} \left(\frac{1}{4} \right)^{1-\frac{1}{q}} G_3(s, q; g_1(\alpha), g_2(\alpha))$$

where $g_1(\alpha), g_2(\alpha)$ is the same as in (2.2), and

$$\begin{aligned} & G_3(s, q; g_1(\alpha), g_2(\alpha)) \\ &= |f'(b)|^s \left[[g_1(\alpha(sq, sq))]^{\frac{1}{q}} + [g_2(\alpha(sq, sq))]^{\frac{1}{q}} \right], \quad |f'(a)| \leq 1 \end{aligned}$$

Proof. Since $|f'|^q$ is s -geometrically convex and monotonically decreasing on $[a, b]$, from Lemma 1 and well known power mean inequality, we have

$$\begin{aligned} & \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \\ & \leq \frac{b-a}{2} \int_0^1 |1-2t| |f'(ta + (1-t)b)| dt \\ & \leq \frac{b-a}{2} \left[\int_0^{\frac{1}{2}} (1-2t) |f'(ta + (1-t)b)| dt + \int_{\frac{1}{2}}^1 (2t-1) |f'(ta + (1-t)b)| dt \right] \\ & \leq \frac{b-a}{2} \left[\left(\int_0^{\frac{1}{2}} (1-2t) dt \right)^{1-\frac{1}{q}} \left[\int_0^{\frac{1}{2}} (1-2t) |f'(ta + (1-t)b)|^q dt \right]^{\frac{1}{q}} \right. \\ & \quad \left. + \left(\int_{\frac{1}{2}}^1 (2t-1) dt \right)^{1-\frac{1}{q}} \left[\int_{\frac{1}{2}}^1 (2t-1) |f'(ta + (1-t)b)|^q dt \right]^{\frac{1}{q}} \right] \\ & \leq \frac{b-a}{2} \left(\frac{1}{4} \right)^{1-\frac{1}{q}} \left[\left[\int_0^{\frac{1}{2}} (1-2t) |f'(a^t b^{1-t})|^q dt \right]^{\frac{1}{q}} \right. \\ & \quad \left. + \left[\int_{\frac{1}{2}}^1 (2t-1) |f'(a^t b^{1-t})|^q dt \right]^{\frac{1}{q}} \right] \\ & \leq \frac{b-a}{2} \left(\frac{1}{4} \right)^{1-\frac{1}{q}} \left[\left[\int_0^{\frac{1}{2}} (1-2t) |f'(a)|^{qt^s} |f'(b)|^{q(1-t)^s} dt \right]^{\frac{1}{q}} \right. \\ (2.13) \quad & \left. + \left[\int_{\frac{1}{2}}^1 (2t-1) |f'(a)|^{qt^s} |f'(b)|^{q(1-t)^s} dt \right]^{\frac{1}{q}} \right] \end{aligned}$$

If $|f'(a)| \leq 1$, by (2.3), we get that

$$\begin{aligned}
 & \int_0^{\frac{1}{2}} (1-2t) |f'(a)|^{qt^s} |f'(b)|^{q(1-t)^s} dt \\
 & \leq \int_0^{\frac{1}{2}} (1-2t) |f'(a)|^{sq t} |f'(b)|^{sq(1-t)} dt = |f'(b)|^{sq} g_1(\alpha(sq, sq)) \\
 (2.14) \quad & \int_{\frac{1}{2}}^1 (2t-1) |f'(a)|^{qt^s} |f'(b)|^{q(1-t)^s} dt \\
 & \leq \int_{\frac{1}{2}}^1 (2t-1) |f'(a)|^{sq t} |f'(b)|^{sq(1-t)} dt = |f'(b)|^{sq} g_2(\alpha(sq, sq))
 \end{aligned}$$

By combining of (2.13)-(2.14) immediately gives the required inequality (2.12). \square

Corollary 2. Let $f : I \subseteq (0, \infty) \rightarrow (0, \infty)$ be differentiable on I° , $a, b \in I$ with $a < b$, and $f' \in L([a, b])$. If $|f'|^q$ is s -geometrically convex and monotonically decreasing on $[a, b]$ for $q \geq 1$, and $s \in (0, 1]$, then

i) If we take $q = 1$ in (2.12), we obtain that

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{2} G_3(s, 1; g_1(\alpha), g_2(\alpha))$$

ii) If we take $s = 1$ in (2.12), for geometrically convex, we obtain that

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{b-a}{2} \left(\frac{1}{4} \right)^{1-\frac{1}{q}} G_3(1, q; g_1(\alpha), g_2(\alpha))$$

where $g_1(\alpha), g_2(\alpha), \alpha(u, v), G_3(s, q; g_1(\alpha), g_2(\alpha))$ are same with above.

3. APPLICATIONS TO SOME SPECIAL MEANS

Let

$$\begin{aligned}
 A(a, b) &= \frac{a+b}{2}, \quad L(a, b) = \frac{b-a}{\ln b - \ln a} \quad (a \neq b), \\
 L_p(a, b) &= \left(\frac{b^{p+1} - a^{p+1}}{(p+1)(b-a)} \right)^{1/p}, \quad a \neq b, \quad p \in \mathbb{R}, \quad p \neq -1, 0
 \end{aligned}$$

be the arithmetic, logarithmic, generalized logarithmic means for $a, b > 0$ respectively.

Proposition 1. Let $0 < a < b \leq 1$, $0 < s < 1$. Then

$$\begin{aligned}
 & |A(a^s, b^s) - [L_s(a, b)]^s| \\
 & \leq (3.1) \frac{(b-a)s b^{s(s-1)}}{2} L\left(a^{s(s-1)}, b^{s(s-1)}\right) \left[A\left(a^{s(s-1)}, b^{s(s-1)}\right) - (1/2) L\left(a^{s(s-1)}, b^{s(s-1)}\right) \right]
 \end{aligned}$$

Proof. The proof is obvious from Theorem 3 applied $f(x) = x^s/s$, $x \in (0, 1]$, $0 < s < 1$. Then $|f'(a)| = a^{s-1} > b^{s-1} = |f'(b)| \geq 1$ and

$$(3.2) \quad \left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x) dx \right| = \frac{1}{s} |A(a^s, b^s) - [L_s(a, b)]^s|,$$

$$\begin{aligned}
 & |f'(\alpha(s, s))g_1(\alpha(s, s)) + g_2(\alpha(s, s))| \\
 = & b^{s(s-1)} \frac{4\sqrt{\left(\frac{a}{b}\right)^{s(s-1)}} - \ln\left(\frac{a}{b}\right)^{s(s-1)} - 2\left(\frac{a}{b}\right)^{s(s-1)} + \left(\frac{a}{b}\right)^{s(s-1)} \ln\left(\frac{a}{b}\right)^{s(s-1)} - 2}{\left[\ln\left(\frac{a}{b}\right)^{s(s-1)}\right]^2} \\
 = & \frac{b^{s(s-1)}}{\ln a^{\frac{s-1}{s}} - \ln b^{\frac{s-1}{s}}} \left(\frac{a^{s(s-1)} - b^{s(s-1)}}{b^{s(s-1)}} \right) \left[\frac{a^{s(s-1)} + b^{s(s-1)}}{2b^{s(s-1)}} - \frac{1}{2b^{s(s-1)}} \frac{a^{s(s-1)} - b^{s(s-1)}}{\ln a^{s(s-1)} - \ln b^{s(s-1)}} \right] \\
 = & b^{s(s-1)} L\left(a^{s(s-1)}, b^{s(s-1)}\right) \left[A\left(a^{s(s-1)}, b^{s(s-1)}\right) - (1/2) L\left(a^{s(s-1)}, b^{s(s-1)}\right) \right].
 \end{aligned}$$

From (3.2) and (3.3), we have the desired inequality. \square

Proposition 2. Let $0 < a < b \leq 1$, $0 < s < 1$. Then

$$(3.4) \quad |A(a^s, b^s) - [L_s(a, b)]^s| \leq \frac{(b-a)sb^{sq(1-s)}}{2(p+1)^{1/p}} \left[L\left(a^{sq(s-1)}, b^{sq(s-1)}\right) \right]^{1/q}$$

Proof. The proof is obvious from Theorem 4 applied $f(x) = x^s/s$, $x \in (0, 1]$, $0 < s < 1$ and $q > 1$. Then $|f'(a)| = a^{s-1} > b^{s-1} = |f'(b)| \geq 1$ and

$$(3.5) \quad g_3(\alpha(sq, sq)) = \frac{a^{sq(s-1)} - b^{sq(s-1)}}{b^{sq(s-1)}(\ln a^{sq(s-1)} - \ln b^{sq(s-1)})} = \frac{1}{b^{sq(s-1)}} L\left(a^{sq(s-1)}, b^{sq(s-1)}\right)$$

From (3.5), we have the desired inequality. \square

Proposition 3. Let $0 < a < b \leq 1$, $0 < s < 1$ and $q \geq 1$. Then

$$(3.6) \quad |A(a^s, b^s) - [L_s(a, b)]^s| \leq \frac{s(b-a)}{2} \left(\frac{1}{4}\right)^{1-\frac{1}{q}} b^{s(s-1)} \left[U^{\frac{1}{q}} + V^{\frac{1}{q}}\right]$$

Proof. The proof is obvious from Theorem 5 applied $f(x) = x^s/s$, $x \in (0, 1]$, $0 < s < 1$ and $q > 1$. Then $|f'(a)| = a^{s-1} > b^{s-1} = |f'(b)| \geq 1$ and

$$(3.7) \quad g_1(\alpha(sq, sq)) = U = \frac{1}{\ln a^{sq(s-1)} - \ln b^{sq(s-1)}} \left(\frac{1}{b^{\frac{sq(s-1)}{2}}} L\left(a^{\frac{sq(s-1)}{2}}, b^{\frac{sq(s-1)}{2}}\right) - 1 \right),$$

$$\begin{aligned}
 (3.8) \quad g_2(\alpha(sq, sq)) &= V = \frac{\left(\frac{a}{b}\right)^{2qs(s-1)}}{(\ln a^{sq(s-1)} - \ln b^{sq(s-1)})} \times \\
 &\quad \left[1 - \frac{\left(\frac{a}{b}\right)^{sq(s-1)} + 1}{\left(\frac{a}{b}\right)^{sq(s-1)} \left(\ln a^{\frac{sq(s-1)}{2}} - \ln b^{\frac{sq(s-1)}{2}} \right)} \right]
 \end{aligned}$$

From (3.7) and (3.8), we have the desired inequality. \square

REFERENCES

- [1] H. Hudzik and L. Maligranda: Some remarks on s -convex functions, Aequationes Math., Vol. 48 (1994), 100–111.
- [2] T.-Y. Zhang, A.-P. Ji and F. Qi: On Integral inequalities of Hermite-Hadamard Type for s -Geometrically Convex Functions. Abstract and Applied Analysis. doi:10.1155/2012/560586.
- [3] B.-Y. Xi, R.-F. Bai and F. Qi: Hermite-Hadamard type inequalities for the m - and (α, m) -geometrically convex functions. Aequationes Math., doi: 10.1007/s00010-011-0114-x.
- [4] S.S. Dragomir, R.P. Agarwal: Two inequalities for differentiable mappings and applications to special means of real numbers and to trapezoidal formula. Appl Math Lett, Vol. 11 No:5, (1998) 91–95.
- [5] J. Hadamard: Étude sur les propriétés des fonctions entières et en particulier d'une fonction considérée par Riemann, J. Math Pures Appl., 58, (1893) 171–215.
- [6] M. E. Özdemir, M. Tunç, and A. O. Akdemir: On $(h-s)_{1,2}$ -convex functions and Hadamard-type inequalities, <http://arxiv.org/abs/1201.6138> (submitted)
- [7] M. Tunç: On some new inequalities for convex fonctions, Turk.J.Math. 36 (2012), 245-251.
- [8] D. S. Mitrinović, J. Pečarić and A. M. Fink: Classical and new inequalities in analysis, KluwerAcademic, Dordrecht, 1993.
- [9] S. S. Dragomir and C. E. M. Pearce: Selected topics on Hermite-Hadamard inequalities and applications, RGMIA monographs, Victoria University, 2000. [Online: <http://www.staff.vu.edu.au/RGMIA/monographs/hermite-hadamard.html>].
- [10] J. E. Pečarić, F. Proschan and Y. L. Tong: Convex Functions, Partial Orderings, and Statistical Applications, Academic Press Inc., 1992.

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